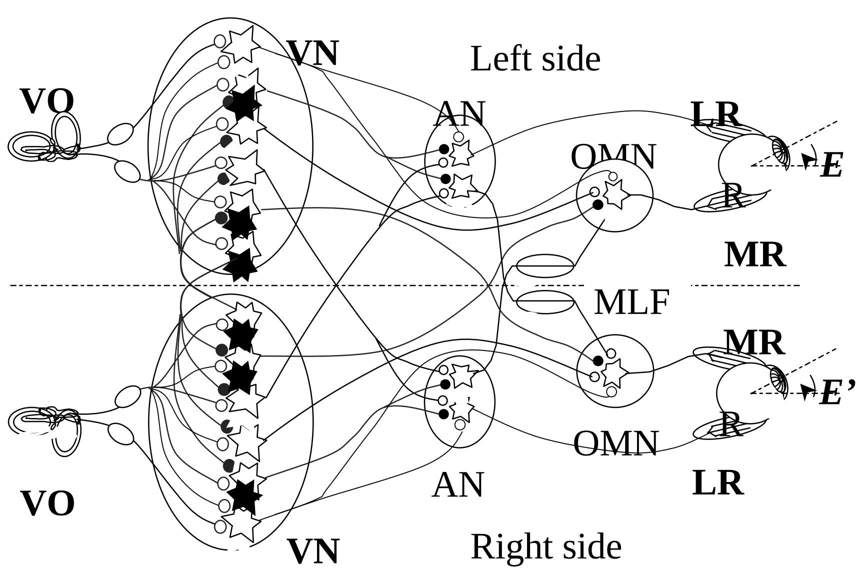
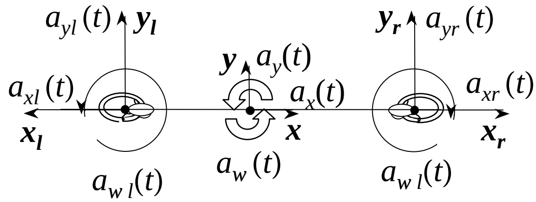
両眼前庭動眼反射の数学モデル





$a_{wl}(t) = a_w(t)$

$a_{xl}(t) = -rw^2(t) - a_x(t)$

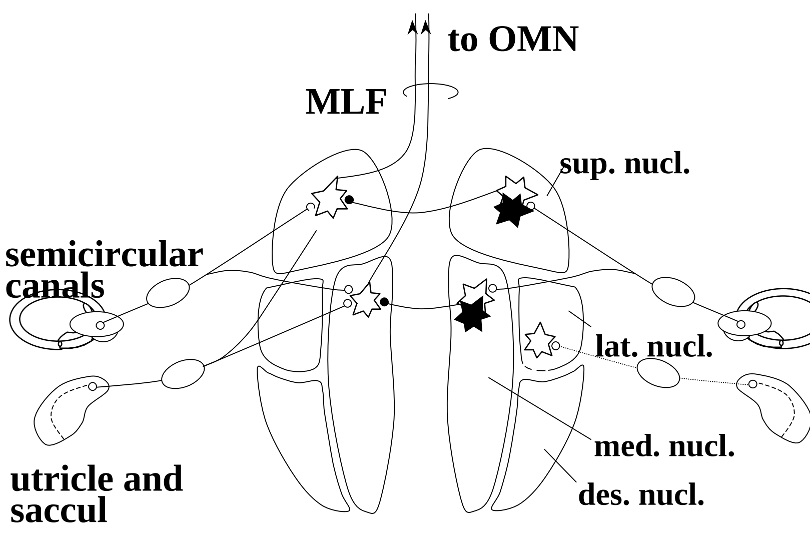
$a_{yl}(t) = -ra_w(t) + a_y(t)$

$a_{wr}(t) = -a_w(t)$

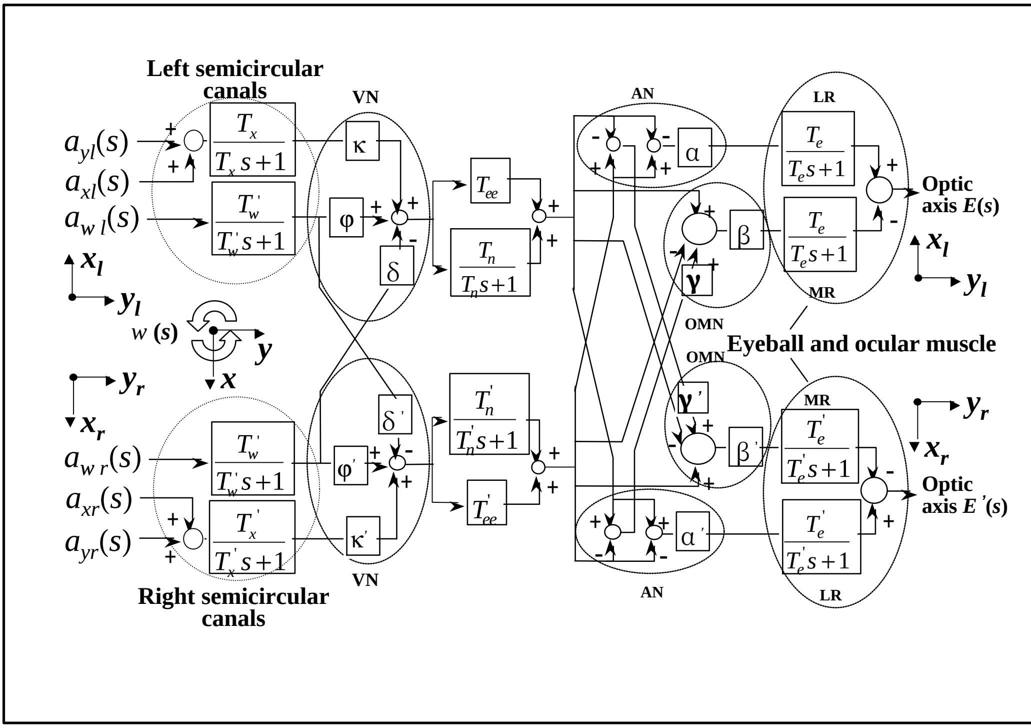
$a_{xr} = -rw^2(t) + a_x(t)$

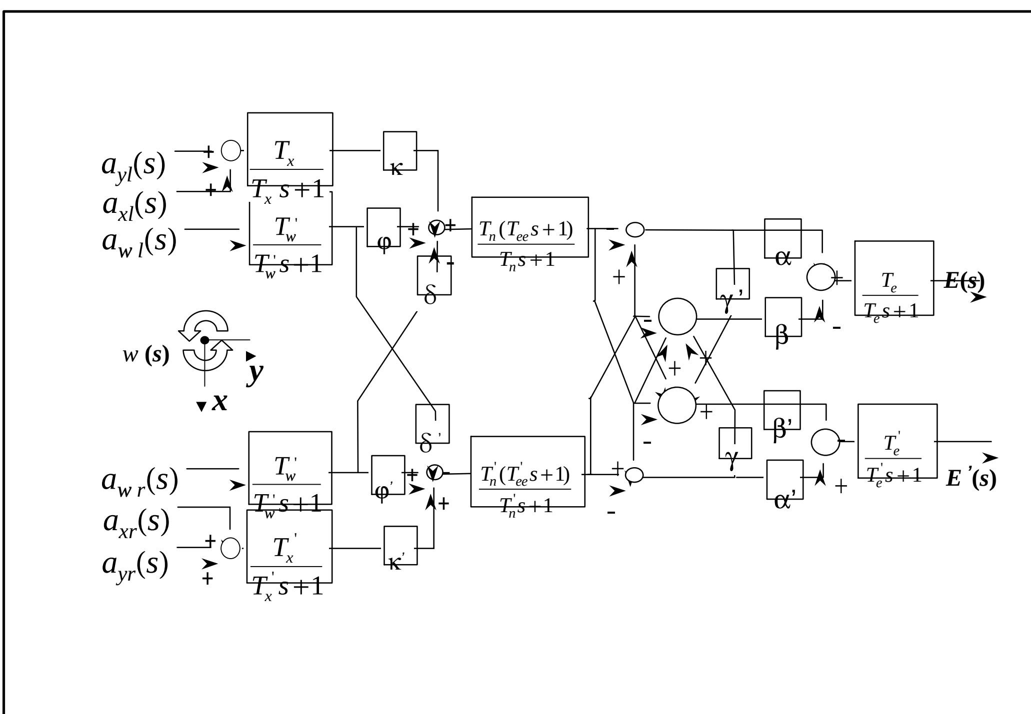
$a_{yr} = ra_w(t) + a_y(t)$

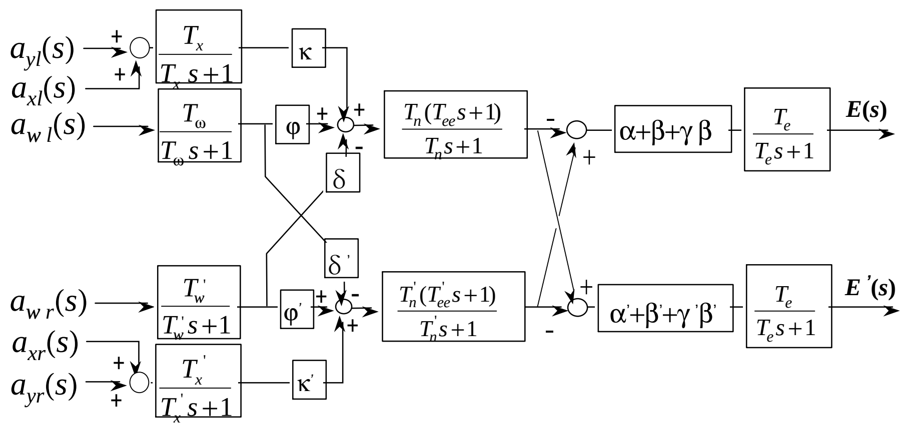
頭部と左右前庭器の座標設定



前庭器～前庭核～動眼神経核の神経経路


Left semicircular canals
VN
AN
LR
MR
OMN
Eyeball and ocular muscle
Optic axis E(s)
Optic axis E '(s)
w (s)
Right semicircular canals


w (s)
E(s)
E '(s)



$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \begin{bmatrix} \frac{T_e}{T_e s+1} & 0 \\ 0 & \frac{T_e'}{T_e' s+1} \end{bmatrix} \begin{bmatrix} -(\alpha+\beta+\beta\gamma) & \alpha+\beta+\beta\gamma \\ \alpha'+\beta'+\beta'\gamma' & -(\alpha'+\beta'+\beta'\gamma') \end{bmatrix} \begin{bmatrix} \frac{T_n(T_{ee}s+1)}{T_n s+1}\left\{\frac{\kappa T_x}{T_x s+1}\left(a_{xl}(s)+a_{yl}(s)\right)+\frac{\varphi T_\omega}{T_\omega s+1}a_{\omega l}(s)-\frac{\delta T_\omega'}{T_\omega' s+1}a_{\omega r}(s)\right\} \\ \frac{T_n'(T_{ee}'s+1)}{T_n' s+1}\left\{\frac{\kappa' T_x'}{T_x' s+1}\left(a_{xr}(s)+a_{yr}(s)\right)+\frac{\varphi' T_\omega'}{T_\omega' s+1}a_{\omega r}(s)-\frac{\delta' T_\omega}{T_\omega s+1}a_{\omega l}(s)\right\} \end{bmatrix}$$

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \begin{bmatrix} \frac{-T_e(\alpha+\beta+\beta\gamma)}{T_e s+1} & \frac{T_e(\alpha+\beta+\beta\gamma)}{T_e s+1} \\ \frac{T_e'(\alpha'+\beta'+\beta'\gamma')}{T_e' s+1} & \frac{-T_e'(\alpha'+\beta'+\beta'\gamma')}{T_e' s+1} \end{bmatrix} \begin{bmatrix} \frac{T_n(T_{ee}s+1)}{T_n s+1}\left\{\frac{\kappa T_x}{T_x s+1}\left(a_{xl}(s)+a_{yl}(s)\right)+\frac{\varphi T_\omega}{T_\omega s+1}a_{\omega l}(s)-\frac{\delta T_\omega'}{T_\omega' s+1}a_{\omega r}(s)\right\} \\ \frac{T_n'(T_{ee}'s+1)}{T_n' s+1}\left\{\frac{\kappa' T_x'}{T_x' s+1}\left(a_{xr}(s)+a_{yr}(s)\right)+\frac{\varphi' T_\omega'}{T_\omega' s+1}a_{\omega r}(s)-\frac{\delta' T_\omega}{T_\omega s+1}a_{\omega l}(s)\right\} \end{bmatrix}$$

T_w>>t ; T_x>>t ; T_n>>t ; T_{ee}= T_e　両側が完全対称する場合

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -(a+b+bg) & (a+b+bg) \\ (a'+b'+b'g') & -(a'+b'+b'g') \end{bmatrix} \begin{bmatrix} k\left(a_{xl}(s)+a_{yl}(s)\right)+j\,a_{wl}(s)-da_{wr}(s) \\ k'\left(a_{xr}(s)+a_{yr}(s)\right)+j'\,a_{wr}(s)-d'\,a_{wl}(s) \end{bmatrix}$$

また,　$a_{xl}(t)= -rw^2(t)-a_x(t)$　　　$a_{xr}= -rw^2(t)+a_x(t)$

$a_{yl}(t) =-ra_w(t)+a_y(t)$　　　$a_{yr}=ra_w(t)+a_y(t)$

$a_{wl}(t) =a_w(t)$　　　$a_{wr}(t) =-a_w(t)$

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -(a+b+bg) & (a+b+bg) \\ (a'+b'+b'g') & -(a'+b'+b'g') \end{bmatrix} \begin{bmatrix} ka_x(s)-(j+d+kr)a_w(s) \\ -k'a_x(s)+(j'+d'+k'r)a_w(s) \end{bmatrix}$$

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{2T_e(a + b + bg)}{s^2} \begin{bmatrix} ka_x(s) - (j + d - kr)a_w(s) \\ -ka_x(s) + (j + d - kr)a_w(s) \end{bmatrix}$$

正常時の眼球運動システムの入出力関係

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{2T_e(a + b + bg)}{s^2} \begin{bmatrix} ka_x(s) - (j + d)a_w(s) \\ -ka_x(s) + (j + d)a_w(s) \end{bmatrix}$$

LVNが損傷した場合 :φ=0, δ=0, κ=0, δ'=0

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -(a + b + bg) & (a + b + bg) \\ (a'+b'+b'g') & -(a'+b'+b'g') \end{bmatrix} \begin{bmatrix} 0 \\ -k'a_x(s) + j'a_w(s) \end{bmatrix}$$

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e(a + b + bg)}{s^2} \begin{bmatrix} ka_x(s) - j\,a_w(s) \\ -ka_x(s) + j\,a_w(s) \end{bmatrix}$$

LANが損傷した場合 :α=0, γ'=0

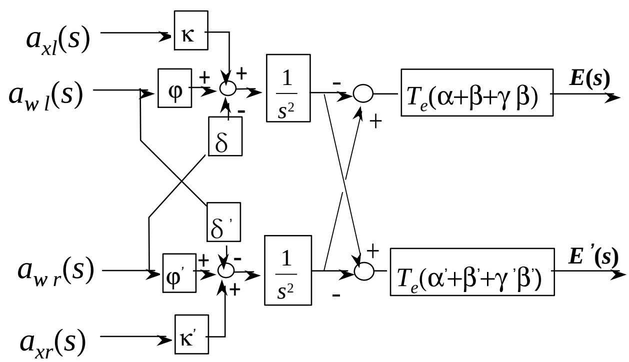
$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -(b + bg) & (b + bg) \\ (a'+b') & -(a'+b') \end{bmatrix} \begin{bmatrix} ka_x(s) - (j + d + kr)a_w(s) \\ -k'a_x(s) + (j'+d'+k'r)a_w(s) \end{bmatrix}$$

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -(b + bg)\{ka_x(s) - (j + d)a_w(s)\} \\ (a + b)\{k'a_x(s) - (j + d)a_w(s)\} \end{bmatrix}$$

LOMNが損傷した場合 :β=0, γ=0

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -a & a \\ (a'+b'+b'g') & -(a'+b'+b'g') \end{bmatrix} \begin{bmatrix} ka_x(s) - (j + d + kr)a_w(s) \\ -k'a_x(s) + (j'+d'+k'r)a_w(s) \end{bmatrix}$$

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e}{s^2} \begin{bmatrix} -a\{ka_x(s) - (j + d)a_w(s)\} \\ (a + b + bg)\{k'a_x(s) - (j + d)a_w(s)\} \end{bmatrix}$$



LVNが損傷した場合 :φ=0, δ=0, δ'=0

if :α= α', β=β', γ=γ' :

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e T_n T_v (T_{ee}s+1)s^2}{(T_e s+1)(T_n s+1)(T_v s+1)} \begin{bmatrix} -0 & (a+b+g\,b)j' \\ 0 & -(a'+b'+g'b')j' \end{bmatrix} \begin{bmatrix} H_{pf}(s) \\ H'_{pf}(s) \end{bmatrix}$$

LANが損傷した場合 :α=0, γ'=0

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e T_n T_v (T_{ee}s+1)s^2}{(T_e s+1)(T_n s+1)(T_v s+1)} \begin{bmatrix} -(b+g\,b)(j+d') & (b+g\,b)(j'+d) \\ (a'+b')(j+d') & -(a'+b')(j'+d) \end{bmatrix} \begin{bmatrix} H_{pf}(s) \\ H'_{pf}(s) \end{bmatrix}$$

LOMNが損傷した場合 :β=0, γ=0

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = \frac{T_e T_n T_v (T_{ee}s+1)s^2}{(T_e s+1)(T_n s+1)(T_v s+1)} \begin{bmatrix} -a(j+d') & a(j'+d) \\ (a'+b'+g'b')(j+d') & -(a'+b'+g'b')(j'+d) \end{bmatrix} \begin{bmatrix} H_{pf}(s) \\ H'_{pf}(s) \end{bmatrix}$$

If $t \ll T_n$, T_v, and $T_{ee}=T_e$, φ= φ', δ= δ', α= α', β=β', γ=γ'

$$\begin{bmatrix} E(s) \\ E'(s) \end{bmatrix} = T_e(a+b+g\,b)(j+d) \begin{bmatrix} -1 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} H_p(s) \\ H'_p(s) \end{bmatrix} = 2rT_e(a+b+g\,b)(j+d)H_r(s)$$